**Tomography**

May 23, 2025

As we see in the class, Radon transform is the projection of an object obtained by line integrals. Hence, it is well suited for ray optics. Please visit the Wikipedia page for nice visualizations: <https://en.wikipedia.org/wiki/Radon_transform>

Modern numerical libraries provide tools to compute the Radon transform, which converts a 2D object into a series of 1D projections, and its inverse, which reconstructs the 2D object from these projections. The collection of these 1D projections, when stacked together, forms a 2D image known as a sinogram (see Figure 1 for a conceptual example). Applying the inverse Radon transform to this sinogram allows us to recover an image of the original object, referred to as a reconstruction (conceptually shown in Figure 2).

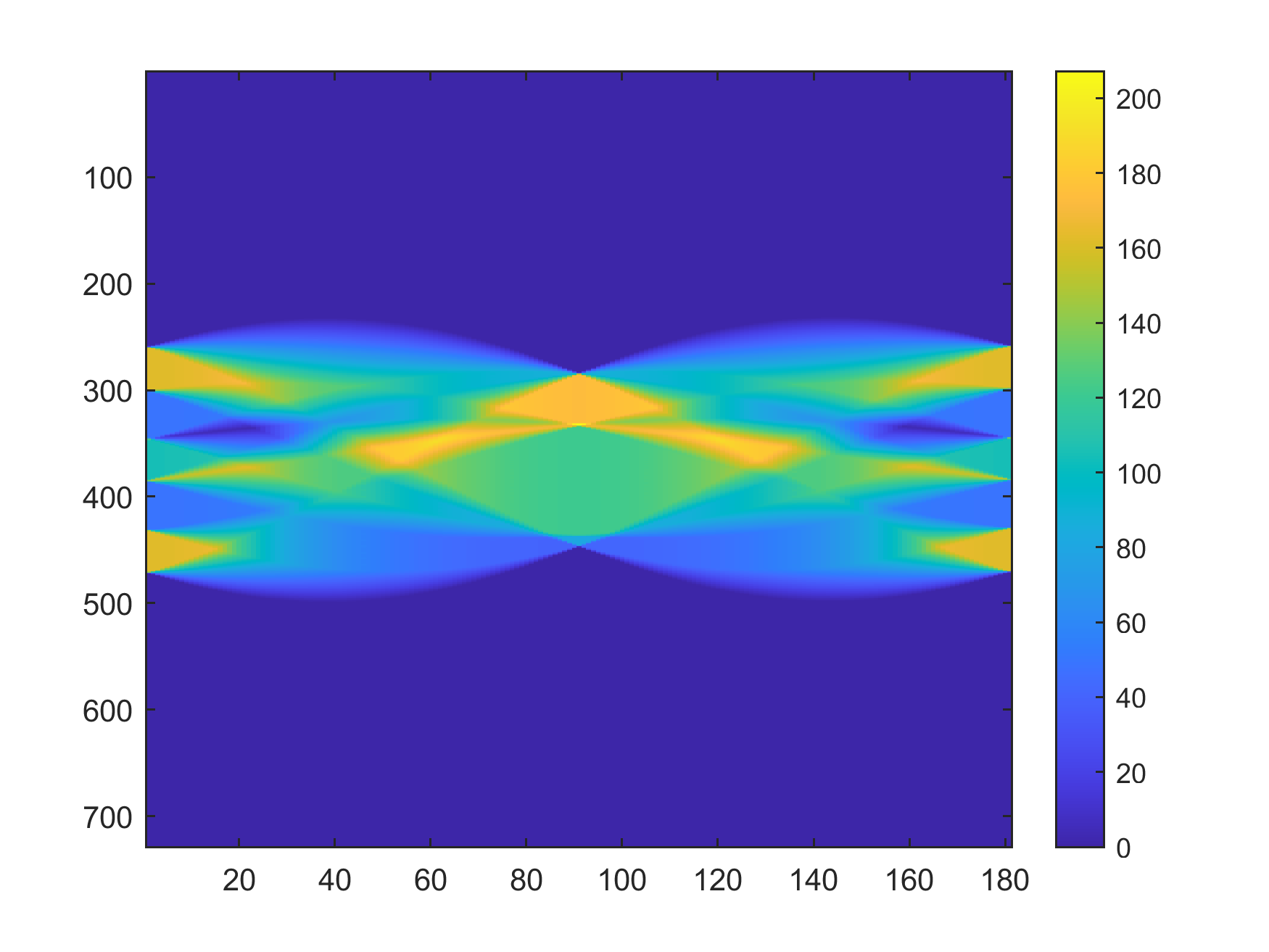
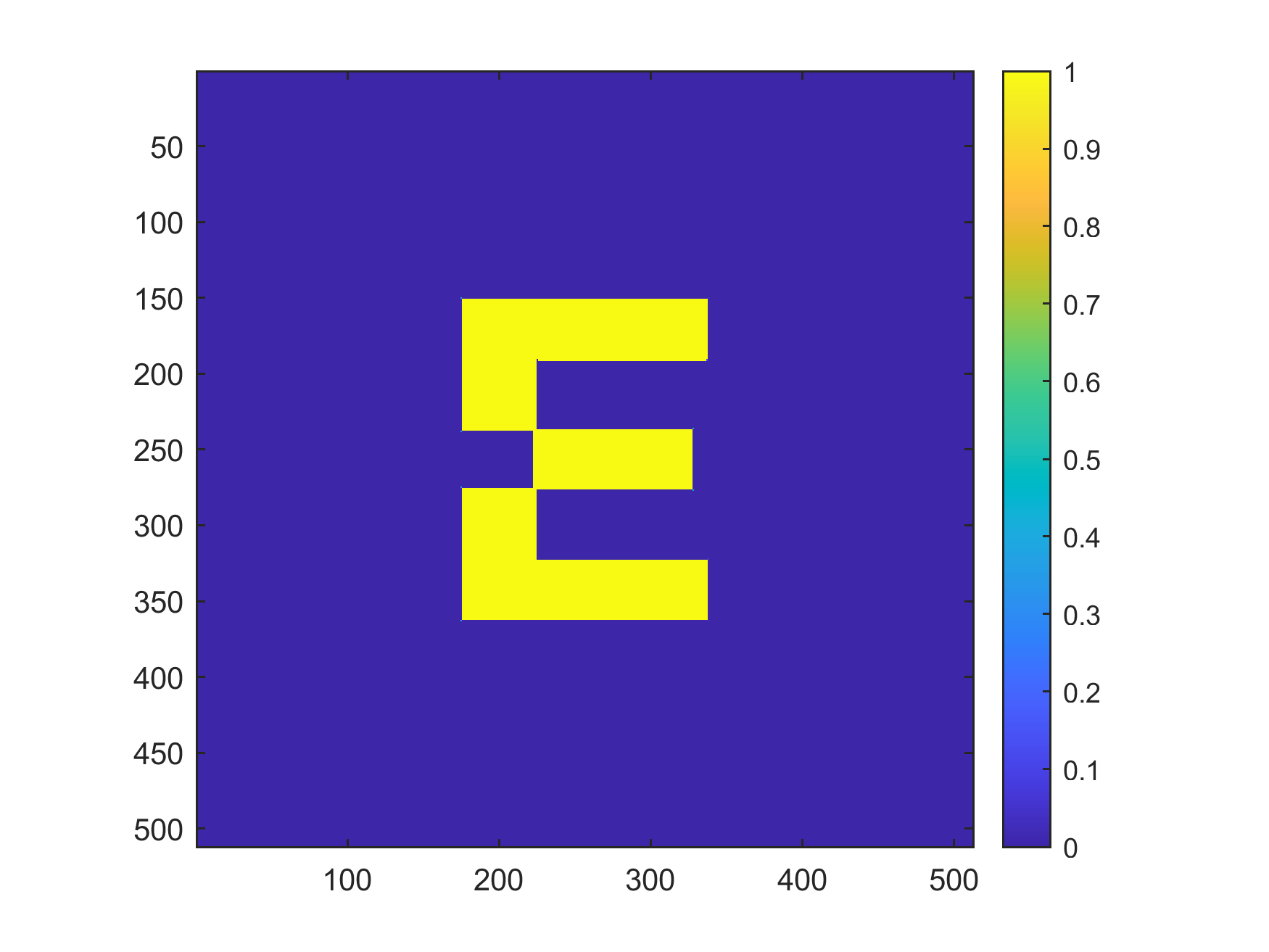


Figure 1: (Left) The image, (Right) Radon transform by using -90° to 90° with 1° steps

When we apply inverse Radon transform to the sinogram we have in Figure 1, we obtain the following result, which we call as a *reconstruction* of the object:

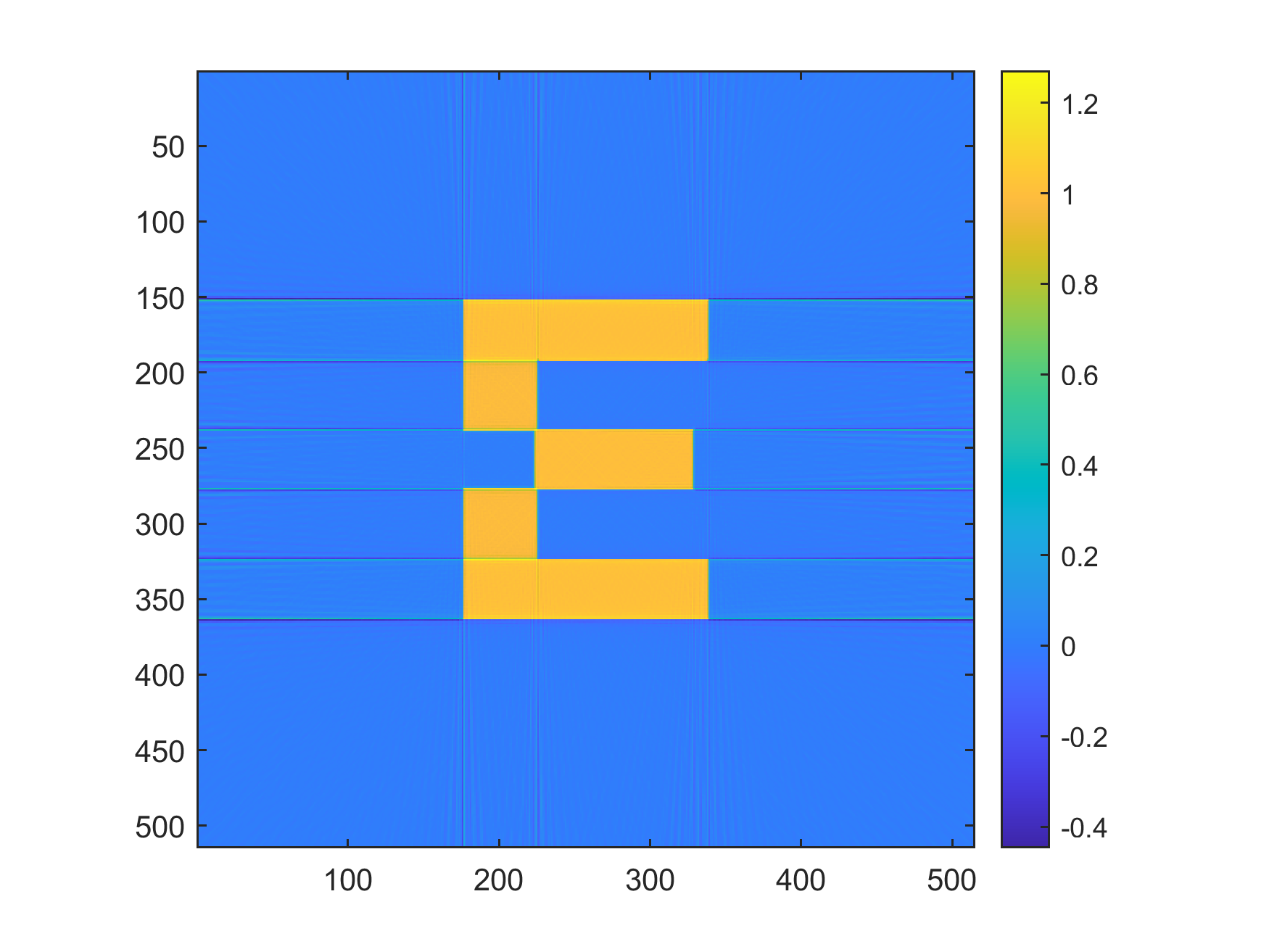


Figure 2: Reconstruction by inverse Radon transform from 180° range sinogram.

**PART A**

**1.** Apply Radon transform to letter E (Provided in Moodle) with 60° (theta=-30:30) and 120° (theta=-60:60) angle ranges with 1° steps and 180° angle range with 10° steps (theta=-90:10:90). Apply inverse Radon transform to obtained sinograms. What do you observe?

**2.** Apply Radon transform to letter E with projections from -90° to -30°, -30° to 30° and 30° to 90°. Apply inverse Radon transform to obtained sinograms. What do you observe when you add up the reconstructions?

**3.** In practice, especially in optical tomography or when wave effects are considered, projections might not be simple line integrals. We can use a 2D Beam Propagation Method (BPM) to simulate the propagation of a wavefront through a 2D object and record the 1D field profile (projection) at the output. While practical tomography often involves 2D projections of 3D objects, this exercise simplifies it to 1D projections of a 2D object. In this exercise, use the letter E as your transparent object immersed in water. Set its refractive index difference with respect to the water as 110-4, and record 1D projections with the provided 2D BPM in Moodle with an angle range and step size of your choice. Then, use the sinograms obtained by stacking the 1D projections in either phase or amplitude to apply inverse Radon transform to get the reconstruction.

Simulation parameters:

Nx=512;

Nz=512;

Lx\_=6e-3;

Lz\_=6e-3;

n0\_=1.34; % Refractive index of the background medium (water)

lambda0\_=532e-9;

delta\_n=1e-4; % Refractive index difference of the object with respect to background medium

**4.** Repeat part 3 by reducing the dimension of your computational window to 0.6 mm by 0.6 mm and 0.1 mm by 0.1 mm. What do you observe?

**PART B**

**Iterative Refractive Index Reconstruction using BPM and Gradient Descent**

Standard inverse Radon transforms can be limited, especially when projection data is affected by wave phenomena like diffraction, or when data is incomplete. In this part, you will implement an iterative approach to reconstruct the 2D refractive index distribution (δn(x,y)) of an object.

Initial Guess: Use the reconstruction obtained from the inverse Radon transform of the phase-based sinogram as the initial guess for your object's scaled refractive index distribution.

Set up an optimization loop (e.g., using gradient descent with PyTorch's autograd). The parameters to be optimized are the pixel values of your refractive index guess.

Calculate the Mean Squared Error (MSE) between your complete set of simulated (and cropped) phase projections for the current iteration and your original (ground truth) phase sinogram.

Use this MSE loss to compute gradients with respect to your refractive index guess and update the guess using an optimizer (e.g., Adam).